Full Solutions
MATH105 April 2009

April 16, 2015

How to use this resource

- When you feel reasonably confident, simulate a full exam and grade your solutions. This document provides full solutions that you can use to grade your work.
- If you're not quite ready to simulate a full exam, we suggest you thoroughly and slowly work through each problem. To check if your answer is correct, without spoiling the full solution, we provide a pdf with the final answers only. Download the document with the final answers here.
- Should you need more help, check out the hints and video lecture on the Math Education Resources.

Tips for Using Previous Exams to Study: Exam Simulation

Resist the temptation to read any of the solutions below before completing each question by yourself first! We recommend you follow the guide below.

1. **Exam Simulation:** When you've studied enough that you feel reasonably confident, print the raw exam (click here) without looking at any of the questions right away. Find a quiet space, such as the library, and set a timer for the real length of the exam (usually 2.5 hours). Write the exam as though it is the real deal.
2. **Reflect on your writing:** Generally, reflect on how you wrote the exam. For example, if you were to write it again, what would you do differently? What would you do the same? In what order did you write your solutions? What did you do when you got stuck?
3. **Grade your exam:** Use the solutions in this pdf to grade your exam. Use the point values as shown in the original exam.
4. **Reflect on your solutions:** Now that you have graded the exam, reflect again on your solutions. How did your solutions compare with our solutions? What can you learn from your mistakes?
5. **Plan further studying:** Use your mock exam grades to help determine which content areas to focus on and plan your study time accordingly. Brush up on the topics that need work:
   - Re-do related homework and webwork questions.
   - The Math Education Resources offers mini video lectures on each topic.
   - Work through more previous exam questions thoroughly without using anything that you couldn't use in the real exam. Try to work on each problem until your answer agrees with our final result.
   - Do as many exam simulations as possible.

   Whenever you feel confident enough with a particular topic, move on to topics that need more work. Focus on questions that you find challenging, not on those that are easy for you. Always try to complete each question by yourself first.

This pdf was created for your convenience when you study Math and prepare for your final exams. All the content here, and much more, is freely available on the Math Education Resources.

Question 1 (a) **Easiness: 4.0/5**

SOLUTION. The first step is straightforward partial differentiation. Take the derivative of f with respect to x, treating y as a relative constant (use the chain rule):

$$\frac{\partial f}{\partial x} = \left(\frac{\partial}{\partial x}((1-x)y)\right) e^{(1-x)y} = -ye^{(1-x)y}.$$

Now we just substitute $(x, y) = (2, 1)$:

$$\begin{aligned}\frac{\partial f}{\partial x}(2,1) &= -1e^{(1-2)1} \\ &= -e^{-1}\end{aligned}$$

Question 1 (b) **Easiness: 4.0/5**

SOLUTION. Taking the derivative of $f(x, y) = (2x + y^3)^{10}$ with respect to y using the chain rule gives $\frac{\partial f}{\partial y} = 10(2x + y^3)^9(3y^2)$

Then taking this function's derivative with respect to x using the chain rule gives $\frac{\partial^2 f}{\partial y \partial x} = 90(3y^2)(2x + y^3)^8(2) = 540y^2(2x + y^3)^8$

Question 1 (c) **Easiness: 4.0/5**

SOLUTION. Taking the partial derivatives yields

$$\frac{\partial f}{\partial x} = 2x + y + 3$$

$$\frac{\partial f}{\partial y} = 2y + x$$

Setting these to zero gives

$$\begin{aligned}2x + y + 3 &= 0 \\ x + 2y &= 0\end{aligned}$$

Solving the second equation for x yields x = *-2y*, which we plug into the first equation to obtain $-4y+y+3 = 0$ and hence y = *1*. Thus x = *-2* and so the only point where a maximum or minimum may exist is at $(x, y) = (-2, 1)$.

Question 1 (d) **Easiness: 4.0/5**

SOLUTION. We proceed as in the hints. Let $u = 5 - 2x$ so $du = -2dx$. Substituting gives

$$
\begin{aligned}
\int \frac{7}{u}\left(\frac{du}{-2}\right) &= -\frac{7}{2}\int \frac{du}{u} \\
&= -\frac{7}{2}\ln|u| + C \\
&= -\frac{7}{2}\ln|5-2x| + C \\
&= k\ln|5-2x| + C
\end{aligned}
$$

Thus, $k = -\dfrac{7}{2}$

Question 1 (e)

SOLUTION. No content found.

Question 1 (f)

Easiness: 4.5/5

SOLUTION. We proceed as suggested by the hints. Let $u = x^2 - 4x + 7$ so that $du = (2x-4)dx = 2(x-2)dx$. This gives

$$\int \frac{x-2}{(x^2-4x+7)^2}\,dx = \int \frac{du}{2u^2}$$

Integrating gives

$$\int \frac{du}{2u^2} = \frac{-1}{2u} + C = \frac{-1}{2(x^2-4x+7)} + C$$

Question 1 (g)

Easiness: 4.0/5

SOLUTION. We proceed by integration by parts. Let $u = 5x$ and $dv = \sin(x+1)\,dx$ so that $du = 5$ and $v = -\cos(x+1)$. Then

$$
\begin{aligned}
\int 5x\sin(x+1)\,dx &= 5x(-\cos(x+1)) - \int 5(-\cos(x+1))\,dx \\
&= -5x\cos(x+1) + 5\int \cos(x+1)\,dx \\
&= -5x\cos(x+1) + 5\sin(x+1) + C
\end{aligned}
$$

Question 1 (h)

SOLUTION. No content found.

Question 1 (i)

Solution. **THIS QUESTION HAS NOT YET BEEN REVIEWED! THE SOLUTION BELOW MAY CONTAIN MISTAKES!**

No content found

Question 1 (j)

Easiness: 5.0/5

Solution. This question is asking for the value of

$$\int_0^\infty e^{-2x}\,dx$$

To solve this, we solve the improper integral as follows. First change to a limit

$$\int_0^\infty e^{-2x}\,dx = \lim_{b\to\infty}\int_0^b e^{-2x}\,dx$$

Next we integrate:

$$\lim_{b\to\infty}\int_0^b e^{-2x}\,dx = \lim_{b\to\infty}\left.\frac{e^{-2x}}{-2}\right|_0^b$$

Plug in the endpoints:

$$\lim_{b\to\infty}\left.\frac{e^{-2x}}{-2}\right|_0^b = \lim_{b\to\infty}\left(\frac{e^{-2b}}{-2} - \frac{e^{-2(0)}}{-2}\right)$$

And we notice that the first limit is 0 (e to a large negative number is 0) and the second is independent of the limit and thus

$$\lim_{b\to\infty}\left(\frac{e^{-2b}}{-2} - \frac{e^{-2(0)}}{-2}\right) = \frac{1}{2}$$

Thus

$$\int_0^\infty e^{-2x}\,dx = \frac{1}{2}$$

completing the question.

Question 1 (k)

Solution. No content found.

Question 1 (l)

Solution. No content found.

Question 1 (m)

Solution. No content found.

Question 1 (n)

Solution. No content found.

Question 2

Solution. No content found.

Question 3

Solution. No content found.

Question 4 (a)

Solution. No content found.

Question 4 (b)

Solution. No content found.

Question 5 (a)

Solution. No content found.

Question 5 (b)

Solution. No content found.

Question 5 (c)

Solution. No content found.

Question 6

Solution. No content found.

Good Luck for your exams!

