The University of British Columbia

Final Examination - April 22, 2013

Mathematics 105

All Sections

Closed book examination Time: 2.5 hours

Last Name ________________ **First** __________ **Signature** ________________

Student Number ________ **Section Number** ________ **Instructor** ________________

Special Instructions:

No books, notes, or calculators are allowed. A formula sheet is included.

1a,b,c,d,e		15
1f,g,h,i,j		15
1k,l,m,n		12
2		12
3		12
4		10
5		12
6		12
Total		100

[42] **1. Short-Answer Questions**. Put your answer in the box provided but show your work also. Each question is worth 3 marks, but not all questions are of equal difficulty.

(a) Find an equation of the plane which is parallel to the plane $Q : -2x + y = 3z + 1$ and passes through the point $(-1, 1, 2)$.

Answer:

(b) Find an equation for the level curve of $f(x, y) = 3xy^2 + 2y - 1$ that goes through the point $(1, -2)$.

Answer:

(c) Evaluate $\displaystyle\int_0^{5/2} \frac{dx}{\sqrt{25 - x^2}}$.

Answer:

(d) Fill in the blanks with right, left, or midpoint; an interval; and a value of n.

$\displaystyle\sum_{k=0}^{3} f(1.5 + k) \cdot 1$ is a ____________ Riemann sum for f on the interval [_____, _____] with $n =$ _____.

(e) Evaluate $\displaystyle\int_{-1}^{2} |2x|\,dx$.

Answer:

(f) If $F(x) = \displaystyle\int_0^{\cos x} \frac{1}{t^3+6}\,dt$, find $F'(\pi)$.

Answer:

(g) Find the area of the region bounded by the graph of $f(x) = \dfrac{1}{(2x-4)^2}$ and the x-axis between $x = 0$ and $x = 1$.

Answer:

(h) Evaluate $\displaystyle\int \frac{\ln x}{x^7}\,dx$.

Answer:

(i) Evaluate $\displaystyle\sum_{k=0}^{\infty} \frac{1}{e^k k!}$.

Answer:

(j) Find a bound for the error in approximating $\displaystyle\int_1^5 \frac{1}{x}\,dx$ using Simpson's rule with $n = 4$.
Do not write down Simpson's rule approximation S_4.

Answer:

(k) Find the Maclaurin series for $f(x) = \dfrac{1}{2x-1}$.

Answer:

(l) Let k be a constant. Find the value of k such that $f(x) = ke^{-x}e^{(-e^{-x})}$ is a probability density function on $(-\infty, 1]$.

Answer:

(m) Compute the cumulative distribution function corresponding to the probability density function $f(x) = 3x^{-4}$, $x \geq 1$.

Answer:

(n) Find the expected value of the random variable X whose probability density function is $f(x) = \dfrac{2}{9\sqrt[3]{x}}$, $1 \leq x \leq 4$.

Answer:

Full-Solution Problems. In questions 2 – 6, justify your answers and show all your work.

[12] **2**. This problem contains three numerical series. For each of them, find out whether it converges or diverges. You should provide appropriate justification in order to receive credit.

(a) $\displaystyle\sum_{k=2}^{\infty} \frac{\sqrt[3]{k}}{k^2 - k}$.

(b) $\displaystyle\sum_{k=1}^{\infty} \frac{k^{10}10^k(k!)^2}{(2k)!}$.

2.(c) $\displaystyle\sum_{k=3}^{\infty} \frac{1}{k(\ln k)(\ln \ln k)}$.

[12] **3**.

(a) Solve the following initial value problem:

$$\frac{dy}{dx} = \frac{1}{(x^2+x)y}, \quad y(1) = 2.$$

3.(b) Let $f(0) = 1$, $f(2) = 3$ and $f'(2) = 4$. Calculate $\int_0^4 f''\big(\sqrt{x}\big)\,dx$.

[10] **4**. Let $f(x, y) = xye^{-2x-y}$.

(a) Find all critical points of $f(x, y)$.

4.(b) Classify each critical point you found as a local maximum, a local minimum, or a saddle point of $f(x, y)$.

[12] **5.**

(a) A study conducted at a waste disposal site reveals soil contamination over a region that can be described as the interior of the circle $x^2+y^2 = 16$, where x and y are in miles. In order to build a circular enclosure to contain all polluted territory, the manager of the site wants to find the radius of the smallest circle centered at $(2, 2)$ that contains the entire contamination region. Formulate this as a constrained optimization problem, clearly stating the objective function and the constraint. Note that you do not need to do any computation in part (a).

(b) Use the method of Lagrange multipliers to find the maximum and minimum values of $f(x, y) = 6y - y^3 - 3x^2y$ on the circle $x^2 + y^2 = 4$. A solution that does not use the method of Lagrange multipliers will receive no credit, even if the answer is correct.

Blank page provided for continuation of 5.(b)

[12] **6**. Find the interval of convergence of the following series:

(a)

$$\sum_{k=1}^{\infty} \frac{(x+1)^{2k}}{k^2 9^k}.$$

6.(b)

$$\sum_{k=1}^{\infty} a_k (x-1)^k, \quad \text{where } a_k > 0 \text{ for } k = 1, 2, \cdots$$

and
$$\sum_{k=1}^{\infty} \left(\frac{a_k}{a_{k+1}} - \frac{a_{k+1}}{a_{k+2}} \right) = \frac{a_1}{a_2}.$$

The End