

The University of British Columbia

Final Examination - April 22, 2013

Mathematics 105

All Sections

Closed book examination

Time: 2.5 hours

Last Name \_\_\_\_\_ First \_\_\_\_\_ Signature \_\_\_\_\_

Student Number \_\_\_\_\_ Section Number \_\_\_\_\_ Instructor \_\_\_\_\_

Special Instructions:

No books, notes, or calculators are allowed. A formula sheet is included.

**Senate Policy: Conduct during examinations**

- Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
- Candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no candidate shall be permitted to enter the examination room once the examination has begun.
- Candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
- Candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
  - (a) speaking or communicating with other candidates, unless otherwise authorized;
  - (b) purposely exposing written papers to the view of other candidates or imaging devices;
  - (c) purposely viewing the written papers of other candidates;
  - (d) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
  - (e) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)–(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
- Candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
- Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
- Candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

|            |  |     |
|------------|--|-----|
| 1a,b,c,d,e |  | 15  |
| 1f,g,h,i,j |  | 15  |
| 1k,l,m,n   |  | 12  |
| 2          |  | 12  |
| 3          |  | 12  |
| 4          |  | 10  |
| 5          |  | 12  |
| 6          |  | 12  |
| Total      |  | 100 |

[42] 1. **Short-Answer Questions.** Put your answer in the box provided but show your work also. Each question is worth 3 marks, but not all questions are of equal difficulty.

- (a) Find an equation of the plane which is parallel to the plane  $Q : -2x + y = 3z + 1$  and passes through the point  $(-1, 1, 2)$ .

Answer:

- (b) Find an equation for the level curve of  $f(x, y) = 3xy^2 + 2y - 1$  that goes through the point  $(1, -2)$ .

Answer:

- (c) Evaluate  $\int_0^{5/2} \frac{dx}{\sqrt{25 - x^2}}$ .

Answer:

- (d) Fill in the blanks with right, left, or midpoint; an interval; and a value of  $n$ .

$\sum_{k=0}^3 f(1.5 + k) \cdot 1$  is a \_\_\_\_\_ Riemann sum for  $f$   
on the interval [\_\_\_\_\_, \_\_\_\_\_] with  $n =$  \_\_\_\_\_.

(e) Evaluate  $\int_{-1}^2 |2x| dx$ .

Answer:

(f) If  $F(x) = \int_0^{\cos x} \frac{1}{t^3 + 6} dt$ , find  $F'(\pi)$ .

Answer:

(g) Find the area of the region bounded by the graph of  $f(x) = \frac{1}{(2x - 4)^2}$  and the  $x$ -axis between  $x = 0$  and  $x = 1$ .

Answer:

(h) Evaluate  $\int \frac{\ln x}{x^7} dx$ .

Answer:

(i) Evaluate  $\sum_{k=0}^{\infty} \frac{1}{e^k k!}$ .

Answer:

(j) Find a bound for the error in approximating  $\int_1^5 \frac{1}{x} dx$  using Simpson's rule with  $n = 4$ .  
Do not write down Simpson's rule approximation  $S_4$ .

Answer:

(k) Find the Maclaurin series for  $f(x) = \frac{1}{2x - 1}$ .

Answer:

- (l) Let  $k$  be a constant. Find the value of  $k$  such that  $f(x) = ke^{-x}e^{(-e^{-x})}$  is a probability density function on  $(-\infty, 1]$ .

Answer:

- (m) Compute the cumulative distribution function corresponding to the probability density function  $f(x) = 3x^{-4}$ ,  $x \geq 1$ .

Answer:

- (n) Find the expected value of the random variable  $X$  whose probability density function is  $f(x) = \frac{2}{9\sqrt[3]{x}}$ ,  $1 \leq x \leq 4$ .

Answer:

**Full-Solution Problems.** In questions 2 – 6, justify your answers and show all your work.

[12] **2.** This problem contains three numerical series. For each of them, find out whether it converges or diverges. You should provide appropriate justification in order to receive credit.

(a) 
$$\sum_{k=2}^{\infty} \frac{\sqrt[3]{k}}{k^2 - k}.$$

(b) 
$$\sum_{k=1}^{\infty} \frac{k^{10} 10^k (k!)^2}{(2k)!}.$$

$$2.(c) \sum_{k=3}^{\infty} \frac{1}{k(\ln k)(\ln \ln k)}.$$

[12] **3.**

(a) Solve the following initial value problem:

$$\frac{dy}{dx} = \frac{1}{(x^2 + x)y}, \quad y(1) = 2.$$



3.(b) Let  $f(0) = 1$ ,  $f(2) = 3$  and  $f'(2) = 4$ . Calculate  $\int_0^4 f''(\sqrt{x}) dx$ .

[10] 4. Let  $f(x, y) = xye^{-2x-y}$ .

(a) Find all critical points of  $f(x, y)$ .

- 4.(b) Classify each critical point you found as a local maximum, a local minimum, or a saddle point of  $f(x, y)$ .

[12] 5.

- (a) A study conducted at a waste disposal site reveals soil contamination over a region that can be described as the interior of the circle  $x^2 + y^2 = 16$ , where  $x$  and  $y$  are in miles. In order to build a circular enclosure to contain all polluted territory, the manager of the site wants to find the radius of the smallest circle centered at  $(2, 2)$  that contains the entire contamination region. Formulate this as a constrained optimization problem, clearly stating the objective function and the constraint. Note that you do not need to do any computation in part (a).
- (b) Use the method of Lagrange multipliers to find the maximum and minimum values of  $f(x, y) = 6y - y^3 - 3x^2y$  on the circle  $x^2 + y^2 = 4$ . A solution that does not use the method of Lagrange multipliers will receive no credit, even if the answer is correct.

Blank page provided for continuation of 5.(b)

[12] **6.** Find the interval of convergence of the following series:

(a)

$$\sum_{k=1}^{\infty} \frac{(x+1)^{2k}}{k^2 9^k}.$$

6.(b)

$$\sum_{k=1}^{\infty} a_k (x-1)^k, \quad \text{where } a_k > 0 \text{ for } k = 1, 2, \dots$$

and

$$\sum_{k=1}^{\infty} \left( \frac{a_k}{a_{k+1}} - \frac{a_{k+1}}{a_{k+2}} \right) = \frac{a_1}{a_2}.$$

**The End**