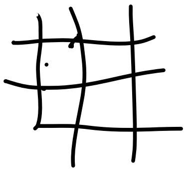
# Lecture 1

January 13, 2017 8:02 PM

Website: http://www.math.ubc.ca/~kliu/common105.html

**1.1 Vectors / xyz - coordinates review**

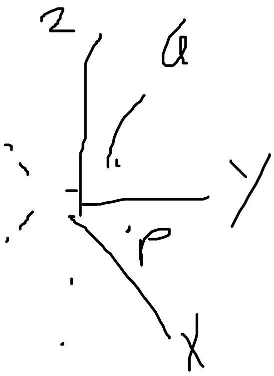
$xy$ - coordinates



(1,2)

Ex: movement of a ship

--> in general, need 3 dimensions for applications
(trajectory of a plane)




Ex $P = (1,0,0)$

$Q = (1,1,1)$

In general, the triple (a,b,c) refers to the point obtained by moving $a$ moving along $x$ axis, distance $b$ along $y$, distance $c$ along $z$

**Vectors**

Vectors represent a translation
- It has magnitude + direction
  - (i.e. "arrow")

We can write vectors as ordered triples

$u = (x, y, z)$

Points = capital, vectors = lowercase

Ex: unit vectors:
- x-unit vector: (1,0,0)
- y-unit vector: (0,1,0)
- z-unit vector: (0,0,1)

Fact

| | |
|---|---|
| Given two points: | $P = (P_1, P_2, P_3)$ |
| | $Q = (Q_1, Q_2, Q_3)$ |

The vector $u = PQ$

$= (Q_1 - P_1, Q_2 - P_2, Q_3 - P_3)$

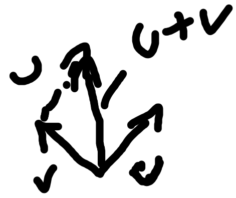
Call $u$ vector from $P \to Q$

<u>Difference between point/vectors</u>

- Vectors correspond to displacement/translation
- Points correspond to locations in space.

| | |
|---|---|
| E.g. | - Times are "Points" (e.g. 4pm)<br>- Durations are translations in time ("+1 hours") |

<u>Note</u> $u = (u_1, u_2, u_3)$ vector is given $u = O \to U$,
Where $O = (0,0,0), u = (U_1, U_2, U_3)$




<u>Vector operations</u>
$u = (u_1, u_2, u_3)$ $v = (v_1, v_2, v_3)$
$u + v = (u_1 + v_1, u_2 + v_2, u_3 + v_3)$

$u - v = u + (-v)$

Flip vector arrow to subtract

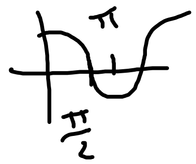
$u - v = (u_1 - v_1, u_2 - v_2, u_3 - v_3)$

<u>Scalar multiplication $\alpha \epsilon \mathbb{R}$</u>
$\alpha * v = (\alpha v_1, \alpha v_2, \alpha v_3)$
(i.e. by scaling by factor $\alpha$)

## 1.2 Dot Product

**Definition:** $u * v = |u||v|cos\theta$
$|u|, |v|$: lengths of vectors $u, v$

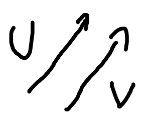
$\theta$: angle between vectors $u, v$
$\theta \epsilon \mathbb{R}(0, \pi]$


$u_1 * v > 0$ since $\theta = \frac{\pi}{4}$ and $\cos\left(\frac{\pi}{4}\right) > 0$
$u_2 * v = 0$ since $\theta = \frac{\pi}{2}$ and $\cos\left(\frac{\pi}{2}\right) = 0$
$u_3 * v < 0$ since $\theta = \frac{3\pi}{4}$ and $\cos\left(\frac{3\pi}{4}\right) < 0$
$u_4 * v = -|u_4||v|$ since $\theta = \pi$ and $\cos(\pi) = -1$

**Definition:** $u_2 v$ are orthoganal
$u * v = 0$ note: zero vector is orthogonal to all other vectors

<u>Parallel vectors</u>

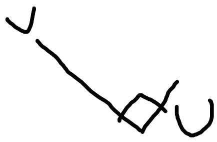



I e θ=0 if U,V point in same direction

i.e. $\theta = 0$ if $u, v$ point in same direction

If $u, v$ are parallel then
$u * v = |u||v|$

Ex
$u = (1,0,0)$
$v = (0,2,0)$




What is $u * v$?
$\quad = 0$ since perpendicular

$u = (1,1,0)$
$v = (1,2,3)$

What is $u * v$?
Problem: need to find angle between $u, v$

Theorem (11.1 in textbook)
$\quad u = (u_1, u_2, u_3) \;\; v = (v_1, v_2, v_3)$
$\quad u * v = u_1 * v_1 + u_2 * v_2 + u_3 * v_3$

E.g. $u = (1,1,0)\,, v = (1,2,3)$
$\quad u * v = 1 * 1 + 1 * 2 + 0 * 3 = 3$

Length/Angle
$u * v = u_1 v_1 + u_2 v_2 + u_3 v_3$
$\quad = |u||v| \cos(\theta) \;\; \theta = angle(u, v)$

$u * u = (u_1)^2 + (u_2)^2 + (u_3)^2 = |u|^2 * \cos(\theta) = |u|^2$
$\quad |u| = \sqrt{(u_1)^2 + (u_2)^2 + (u_3)^2}$

Angle : $\cos(\theta) = \frac{u * v}{|u||v|}$
$\quad$ i.e. we can compute $\theta$ from $u, v$ in components

$$\cos(\theta) = \frac{u_1 v_1 + u_2 v_2 + u_3 v_3}{\left(\sqrt{(u_1)^2 + (u_2)^2 + (u_3)^2}\right)\left(\sqrt{(v_1)^2 + (v_2)^2 + (v_3)^2}\right)}$$

Properties of dot product
(theorem 11.2)
1) $u * v = v * u$
2) $\alpha(u * v) = (\alpha u) * v = u * (\alpha v)$ $where\ \alpha \epsilon \mathbb{R}$
3) $u(v + w) = u * v + u * w$

Proof: follows from theorem 11.1

**1.3 Planes in $\mathbb{R}^3 (xyz - space)$**
**(page 858/854)**

Lines in $\mathbb{R}^2$ (2-d space) - determine by two points
- alternatively, point + vector

Planes in $\mathbb{R}^3$(3-d space) - determined by three points
- alternatively, point + normal vector

Definition: Given a fixed point $P_0$, non-zero normal vector $n$, the set of points $P \epsilon \mathbb{R}^3$ for which $P_0 \rightarrow P$ is is orthogonal to $n$ is called a plane.